VILLANOVA UNIVERSITY DEPT. OF ASTRONOMY AND ASTROPHYSICS

PHOEBE manual

Andrej Prša (Villanova University, PA, USA) Petr Harmanec (Charles University, Prague, CZ)

Adopted for PHOEBE 0.32

13 April 2010

CONTENTS

Contents

1	Introduction	3
	1.1 PHOEBE license	3
2	Heads-up!	5
3	Running PHOEBE for the first time	5
4	Starting a new project in PHOEBE	7
	4.1 Input data	9
	4.2 Passbands	10
	4.3 Data weighting	15
	4.4 Propagating formal errors	17
5	SOLVING THE INVERSE TASK IN PHOEBE	20
	5.1 DIFFERENTIAL CORRECTIONS	20
	5.2 NELDER AND MEAD SIMPLEX	20
	5.3 USING THE SCRIPTER	20
6	OUTPUT OF RESULTS	20

2

1 Introduction

PHOEBE stands for PHysics Of Eclipsing BinariEs. It is a tool for the modeling of eclipsing binary stars based on photometric and spectroscopic (radial velocity) data. PHOEBE is based on the Wilson and Devinney (1971, hereafter WD) code.

PHOEBE is pronounced fee-bee.

The suite consists of three parts:

• The library: phoebe-lib

This is PHOEBE's scientific and computational core. It contains functions and algorithms that are used for eclipsing binary modeling. It is not a stand-alone application. As such, it cannot be run, nor can it be used in any direct fashion. For interaction with the user, a driver is needed. Provided are two drivers: the GUI and the scripter. These are the front-ends the user sees.

• The graphical user interface: phoebe-gui

PHOEBE's graphical user interface (GUI) is a heavily structured frontend for setting parameter values, plotting light and radial velocity curves, invoking the minimizer and overviewing the model results. The GUI is especially suited for the case-by-case analysis. This manual will predominantly focus on the GUI.

• The scripter: phoebe-scripter

The scripter is a terminal-based front-end that features a full-fledged scripting language developed especially for PHOEBE. It requires a somewhat steep learning curve, to the profit of flexibility and the power of scripting. The scripter may prove useful to users with at least superficial experience in working with the GUI and eclipsing binaries in general. It is especially suited for statistical tests and analysis of larger data-sets.

1.1 PHOEBE license

PHOEBE is and will always be free: both free of charge and free in a sense that you may re-use its code in any way you see fit. PHOEBE is released under the GNU General Public Licence. A copy of the licence can be found in the top-level COPYING file of the distribution. To quote the most important part:

The licenses for most software are designed to take away your freedom to share and change it. By contrast, the GNU General Public License is intended to guarantee your freedom to share and change free software—to make sure the software is free for all its users. This General Public License applies to most of the Free Software Foundation's software and to any other program whose authors commit to using it. (Some other Free Software Foundation software is covered by the GNU Library General Public License instead.) You can apply it to your programs, too.

When we speak of free software, we are referring to freedom, not price. Our General Public Licenses are designed to make sure that you have the freedom to distribute copies of free software (and charge for this service if you wish), that you receive source code or can get it if you want it, that you can change the software or use pieces of it in new free programs; and that you know you can do these things.

To protect your rights, we need to make restrictions that forbid anyone to deny you these rights or to ask you to surrender the rights. These restrictions translate to certain responsibilities for you if you distribute copies of the software, or if you modify it.

For example, if you distribute copies of such a program, whether gratis or for a fee, you must give the recipients all the rights that you have. You must make sure that they, too, receive or can get the source code. And you must show them these terms so they know their rights.

We protect your rights with two steps: (1) copyright the software, and (2) offer you this license which gives you legal permission to copy, distribute and/or modify the software.

Also, for each author's protection and ours, we want to make certain that everyone understands that there is no warranty for this free software. If the software is modified by someone else and passed on, we want its recipients to know that what they have is not the original, so that any problems introduced by others will not reflect on the original authors' reputations.

Finally, any free program is threatened constantly by software patents. We wish to avoid the danger that redistributors of a free program will individually obtain patent licenses, in effect making the program proprietary. To prevent this, we have made it clear that any patent must be licensed for everyone's free use or not licensed at all.

2 Heads-up!

PHOEBE is based¹ on the approximation of stellar shapes by equipotentials. Thus, the program does not use common parameters such as semi-amplitudes of the radial-velocity curves or relative photometric radii. Instead, the stellar shapes are characterized by the values of equipotentials, the mass ratio and semi-major axis. Note that, for proximity-distorted stars, the use of equipotentials implies that the mass ratio is affected not only by RVs but also by photometric data.

3 Running PHOEBE for the first time

In this section we will assume that the installation of PHOEBE and its components is done correctly. Upon invocation, PHOEBE looks for the configuration file phoebe.config in the following locations:

- ~/.phoebe-0.32, a hidden directory in your home directory. This corresponds to the current PHOEBE installation;
- ~/.phoebe-0.xy, where xy are all official PHOEBE releases. This corresponds to all legacy PHOEBE installations, and configuration settings will be imported.

If the configuration file is not found in any of the above locations, you will be presented with the following notice window:



Upon clicking on OK, a Settings window will appear. The settings are split into two tabs: Directories and Options. The Base directory is where PHOEBE expects to find the source code; the Defaults directory is a location of the defaults.phoebe file. The purpose of this file is to make permanent

 $^{^1\}mathrm{Please}$ refer to the <code>PHOEBE</code> Scientific Reference for the details on the model formulation.

changes to the default values of parameters that PHOEBE uses upon start. The Working directory is scratch space where PHOEBE outputs temporary files (useful for control and debugging). The Data directory is where PHOEBE expects to find parameter files; this is usually set up as a parent directory with per-star subdirectories, i.e. projects/EBs might be a Data directory that contains subdirectories such as projects/EBs/UWLMi etc. Finally, the PTF directory contains Passband Transmission Functions supported by PHOEBE. We will discuss these at length in §4.2.

O PHOEBE Settings						
Directories Options						
Main Configuration	n					
Base directory:	📄 phoebe 🗸 🗸					
Defaults directory:	📄 phoebe 🗸 🗸					
Working directory:	i tmp					
Data directory:	📄 phoebe 🗸 🗸					
PTF directory:	j⊇ptf ✓					
Limb darkening ta	bles					
O None						
O PHOEBE (2010) tal	bles					
PHOEBE LD direct	ory:					
🔿 Van Hamme (1993	3) tables					
Van Hamme LD dire	ctory: phoebe 🗸					
Kurucz model spectra						
Kurucz model spectra directory:						
	<u>OK</u> <u>S</u> ave <u>C</u> ancel					

Next, you can opt to use precomputed limb darkening (LD) tables; two tables are currently supported, the internal LD tables computed from Castelli and Kurucz (2004)'s NEWODF models, or van Hamme (1993) tables. Finally, as a part of work in progress, you may select a directory with Kurucz's model spectra; these will be used for full-fledged comparison between observed and Kurucz's spectra. This functionality, however, is not yet made available for PHOEBE 0.32 due to insufficient testing.

The defaults for the directories are listed in Table 1. Make sure you change the **Data** directory to something more appropriate, i.e. a subdirectory with write access in your home directory.

4 STARTING A NEW PROJECT IN PHOEBE

Directory:	Default:
Base	/usr/local/share/phoebe
Defaults	/usr/local/share/phoebe/defaults
Working	/tmp
Data	/usr/local/share/phoebe
PTF	/usr/local/share/phoebe/ptf

Table 1: Default PHOEBE directories in the Settings window.

Take some time to review the **Options** tab of the Settings window. Currently there are only two options available, but more will be added as they are implemented.

Once you have set up the directories, either click Save to permanently store the settings in a configuration file, or click OK to use the settings temporarily (they will be lost upon PHOEBE exit).

4 Starting a new project in PHOEBE

This section will guide you through an examplary PHOEBE run that includes setting up and visualizing the data, and initializing the model. Upon a successful initial configuration, you will be presented with the following screen.

С РНОЕВЕ	×			
<u>File</u> <u>Settings</u> <u>H</u> elp				
Open Save LC Plot RV Plot Fitting Settings Quit				
Data Parameters Fitting Plotting	Ð			
Star	Results summary			
Model: Unconstrained binary system	Value			
Decouple secondary luminosities from tempera	atures Ω(L ₁) 3.750000			
LC data	Ω(L ₂) 3.206796			
Active Filename ID Filter Col. 1 Col. 2 Col. 3 Sigma	M ₁ 6.716980			
Ad	M ₂ 6.716980			
Ed	lit R ₁ 0.000000			
Rem	R ₂ 0.000000			
	M _{bol,1} 0.000000			
	M _{bol.2} 0.000000			
Add LC scatter: Poissonian scatter	erate log(g_) 0.000000 ~			
RV data	Fitting summary			
Active Filename ID Filter Col. 1 Col. 2 Col. 3 Sigma	Parameter Value Step Min I			
Ed	lit			
Rem	ove			
Common options				
Independent variable: Phase 🖌 🗆 Bin input data Zero magnitude: 💟 Primary proximity effect	ts			
Passband mode: Interpolation V No. of bins: 100 C Secondary proximity eff	iects			
PHOEBE started. All fields initialized to default values.				

We will be using a detached binary star UW LMi for all examples; the data are distributed with PHOEBE, or you can download them from the following location:

http://phoebe.fiz.uni-lj.si/files/UWLMi.tar.gz

Type in a name of your star in the Binary star name field; note that the title of the PHOEBE window will change to include this name for reference. If you have any prior information on the morphology of your star, select it from the Model drop-down box. The options are:

- **X-Ray binary.** This mode of operation relies on the accurate eclipse duration timing of the compact object available from X-Ray observations, expressed in terms of the phase angle θ .
- Unconstrained binary system. No morphology constraints are imposed.
- **Overcontact binary of W UMa type.** In this mode the following constraints are applied: primary and secondary star potentials are equal; polar temperature of the secondary star is computed by the gravity darkening law of the entire common envelope; gravity darkening, bolometric albedo and limb darkening parameters of both stars are the same.
- **Detached binary.** This mode is for detached binaries with no constraints on the potentials.
- **Overcontact binary not in thermal contact.** In this mode the components are in geometric contact, but not in thermal contact. The primary and secondary potentials are thus constrained to be equal, but the constraints on the gravity darkening, temperatures, albedoes and limb darkening are not applied.
- Semi-detached binary, primary fills the Roche lobe. This mode corresponds to the cases where the primary exactly fills its classical Roche lobe; the only constraint is the value of the primary potential that is set to its Roche lobe value.
- Semi-detached binary, secondary fills the Roche lobe. Similar the the previous mode, only that the secondary fills its Roche lobe, and accord-ingly, the secondary star potential is constrained.
- **Double contact binary.** In this mode both stars are filling their Roche lobes exactly, and both values of the potential are constrained accordingly.

```
# Object:
              UW LMi
# Passband:
              Johnson V
# Source:
              Crni Vrh Observatory, Slovenia
#
# HJD
                     V
                             StDev
2451642.366073
                   8.278
                             0.004
2451642.367797
                   8.288
                             0.004
2451642.369510
                   8.276
                             0.004
```

Table 2: An example of the input data file.

In all but the unconstrained mode, the secondary star luminosity is computed from the radiative properties rather than from the provided secondary luminosity parameters in the Parameters->Luminosities tab. This constraint can be explicitly removed by checking the Decouple secondary luminosities from temperatures option. Typically you do not want to do this unless there is a good reason to not trust the radiative model.

If you are using UW LMi data, select **Detached binary** morphology and make sure that secondary luminosities are *not* decoupled from temperatures.

4.1 Input data

PHOEBE can treat photometric and RV data either as a function of time (typically heliocentric Julian days) or orbital phase. In the current implementation it is possible to use many individual light curves in various photometric passbands but only one set of RVs for the primary and another one for the secondary. If the RV data come from various sources which do not have the same RV zero point, the user has to correct these RV shifts in advance before using such data in PHOEBE. Work is in progress to remove this limitation.

Observational data enter in a free format. Empty lines and white space are handled gracefully. The comments are denoted with a **#** character; anything that follows **#** and spans to the end of the line is ignored. The number of comments is not limited. Observations typically contain 2 or 3 columns: time (or phase), measured quantity (RV in km/s, magnitude or flux) and – optionally – standard deviation or weight of the observation. Table 2 shows an example of the photometric input data.

Note that no matter whether you specify magnitude or flux in the input

	Passband set:	Passbands:
_	Johnson	U, B, V, R, I, J, H, K, L, M, N
	Cousins	R_C, I_C
	Geneva	U, B, B1, B2, V, V1, G
	Strömgren	$u, v, b, y, H_{\beta,n}, H_{\beta,w}$
	SDSS	u,g,r,i,z
	Bessell IR	J, H, K, L, L', M
	HST	complete FOC system
	Hipparcos	B_T, V_T, H_p
	CoRoT	exo, sismo
	Kepler	default

Table 3: Supported passbands in PHOEBE 0.32.

data file, the program internally always uses fluxes. For that reason you need to specify the approximate magnitude of the binary at quarter-phase to have to flux scale in a reasonable numeric range. This parameter is called Zero magnitude appears in the Data tab.

4.2 Passbands

Every light curve has an associated passband with it, and every passband is described by a transmission function (sometimes called response function) that gives the transmission efficiency as function of wavelength. The transmission function will typically be a combination of the filter response function and the optics detection efficiency (i.e. reflective properties of the mirror, CCD quantum efficiency, etc). However, when using standard photometric systems, such as Johnson, Cousins, Sloan, ..., most instrumental effects will be removed by data reduction. Then a passband transmission curve is essentially the same as the filter response function.

The passband effects on the model are extensive and range from proper account for the radiative physics to limb darkening corrections. The computation of these corrections is involved and computationally intensive, that is why PHOEBE comes with precomputed lookup tables for these corrections, for each passband. This implies that PHOEBE can handle only those passbands for which the tables have been precomputed. Table 3 lists all passbands supported by PHOEBE 0.32.

Since the observed features of radiative properties depend on the used passband, a number of model parameters is passband-dependent: primary and secondary passband luminosities, third light, interstellar extinction, limb darkening, etc. On top of that, the observations in each passband may be weighted differently, according to the light level (cf. §4.3 for the details on data weighting). This means that the proper passband selection is crucial for the accurate synthetic light curve computation.

We are working on implementing custom passband support to PHOEBE; in the mean time you can contact us on the phoebe-discuss mailing list and request radiation and limb darkening tables to be computed for your particular passband.

Radial velocity curves are no different, since the observed radiative features depend on the observed wavelength interval. However, spectroscopic data are rarely filtered and the passband transmission function depends primarily on the properties of optics. Fortunately, the effects on the radial velocity curves are limited to photometric corrections and are second order effects, so the synthetic RV curves are not extremely sensitive to the passband selection. What you typically want to do is select a passband that roughly covers the wavelength range of the spectral lines used for RV determination.

§

To add a new observational curve to PHOEBE, click on the Add button in the LC data or the RV data frame. The following input window will appear.

	PHOEBE - Ac	ld LC Data	a	_ — ×
LC data Name:				
Filename:	(None)			
Column 1:	Time (HJD)	Sigma:	0.01000	\$
Column 3:	Standard weight v	Filter:		~
			<u>O</u> K	<u>C</u> ancel

Every observational curve is identified by its name; although PHOEBE does

not enforce that these names be unique, it is good practice to keep them short and informative. An example may be APT:B (2009) for a light curve acquired on the Automated Photoelectric Telescope (APT) in Johnson B band in 2009. Provide the passband name in the Name field. If you omit the name, a default will be applied, which is in the form filter-set:filter, i.e. Johnson:B.

Next, select the corresponding file that contains the data by clicking on the Filename selection button. This will open the dialog window and allow you to browse to the location of your input file. Make sure that the file format conforms to the specifications provided in §4.1. Note that the filenames are stored with their complete paths, so they may reside anywhere on the disk. Once a filename is selected, its contents will be previewed in the white space below.

From the drop-down boxes assign input file columns: Time (HJD) or Phase for the first column, Flux or Magnitude for the second column, and Standard weight, Standard deviation or Unavailable for the third column. If observed data are radial velocities, the second column can be either a Primary RV or a Secondary RV in km/s.

0	PHOEB	E - A0	d LC Data	a	- O X
LC data					
Name: AP	T:B (2009)				
Filename:	UWLMI.APT.B				
Column 1:	Time (HJD)	~	Sigma:	0.03000	\$
Column 2:	Magnitude	~	Filter:	Johnson:B	×
Column 3:	Unavailable	~			
# Object: UW # Filter: John # Source: Au # Season: 20 # # HJD-24500 4502.88146 4502.88278	/ LMi son B tomatic Photoelectr 108-09 000 diff.mag 50 0.713400 30 0.727700	ic Tele	escope		
4502.88410	00 0.727000				×
				<u>O</u> K	<u>C</u> ancel

Next, select an appropriate value for the standard deviation Sigma of the observed curve. If the third column provides Standard weight or is Unavailable, set the value of Sigma to the estimated root mean square (rms) value of the data-set. It is not crucial to get this value completely right, it suffices to get a right order of magnitude for the first cut at the solution. If, on the other hand, the third column provides **Standard deviation**, then you should set the value of **Sigma** to 1.0 – otherwise you would be weighting data points twice in that data-set.

Finally, select a filter according to the discussion in §4.2. Clicking on OK will add the curve to the list of curves in the Data tab.

Continuing with our UW LMi example, add B, V and I_C light curves acquired with APT, and primary and secondary radial velocities by Griffin (2001) The RV data-set in this manual will be switched to Asiago observations as soon as I finish reducing them since they're ours and Griffin's aren't.

0	PHOEBE - UW LMi		_ 0	×
<u>File</u> <u>S</u> ettings <u>H</u> elp				
Open Save LC	E Plot RV Plot Fitting Settings Quit			
Data Parameters Fitting	g Plotting			₩
- Star		Results su	mmary	
Ripper star pame.	Model: Detached binary V	Parameter	Value	Â
bindry star name:	Decouple secondary luminosities from temperatures	Ω(L ₁)	3.750000	
LC data		Ω(L ₂)	3.206796	
Active Filename	ID Filter Col 1 Col 2 Col 3	M ₁	6.716980	
/home/andrei/p	rojects/EBs/2010_LIWLMi/LIWLMi APT B_APT·B (2009)_lohnson/B_Time (HID)_Magnitude_Linava	M ₂	6.716980	
/home/andrei/p	rojects/EBs/2010_UWLMi/UWLMi.APT.V_APT:V (2009) Johnson:V_Time (HJD) Magnitude Unava Edit	R ₁	0.000000	
/home/andrej/pi	rojects/EBs/2010_UWLMi/UWLMi.APT.I APT:Ic (2009) Cousins:I Time (HJD) Magnitude Unava	R ₂	0.000000	U
	Remove	M _{bol 1}	0.000000	
	,	M	0.000000	
Add LC scatter: Pois	ssonian scatter V Sigma: 0.01000 🗘 Seed: 150000000 🗘 Generate	log(g_)	0.000000	~
DV data		Fitting sur	nmary	
RV data		Parameter	Value Step Mi	nt
Active Filename	ID Filter Col. 1 Col. 2 Add			-
/nome/andrej/p	rojects/EBS/2010_UWLMI/UWIMI.gnmin2001.rv1_Gnmin (2001) Rv1_Jonnson:I_Time (HJD)_Primar			
/nome/andrej/pi	rojects/285/2010_0wEMI/dwinnt.ghtmit/2001.fv2_Ghtmit (2001) kv2_johnson:r=hine (HjD)_Second			
	Remove			
<				
Common ontions				
Independent unrichle				
independent variable:	Time (HJD) Din input data Zero magnitude: Primary proximity effects			
Passband mode:	Interpolation V No. of bins: 100 🗘 0.00 🗘 🗹 Secondary proximity effects			
				>
A radial velocity curve ad	lded.			

Once you are done with adding the data, there are two important details: setting the Zero magnitude and setting the independent variable.

PHOEBE differs from other modeling codes in the treatment of light curves that it uses a single zero magnitude for all passbands. This insures that the color index of calibrated light curves is preserved and that *both* temperatures of the binary components may be determined simultaneously. We will talk about color constraining in detail in later sections.

After inspecting the light curve file contents, you can see that the quarterphase magnitudes of the three light curves are ~ 0.7 , ~ 0.5 and ~ 0.3 for B, V and I_C passbands, respectively. Based on this, you should set the zero magnitude to be close to the dimmest quarter-phase magnitude, thus 0.7. Again, you do not need to worry on getting this value to match observations perfectly; an order of magnitude is perfectly sufficient and we set it here to 0.0.

All computations in PHOEBE can be done either in time space or in phase space. If your ephemeris changes (i.e. dP/dt or $d\omega/dt$ are not 0), then you must perform your computation in time space. This is also the case if you want to fit any time dependent parameter. Otherwise you may perform all calculations in phase space, which is numerically more efficient, but the difference on modern computers is negligible. A rule of thumb is: if your data are in time space, do computations in time space; if your data are in phase space or mixed, do computations in phase space. Set the Independent variable parameter accordingly.

The defaults on other parameters in the Data tab are usually sound. In simulations it is sometimes useful to add synthetic scatter to model light curves; this can be achieved by checking the Add LC scatter switch. The scatter can be randomly sampled from the Poisson distribution (scaled with the square root of the flux; this is the default and is appropriate for most cases), from the uniform distribution (option None), or from the low light level distribution (scaled linearly with flux; this is appropriate for faint sources). The absolute size of scatter is governed by the parameter Sigma, while the seed for the random number generator, if exact repeatability is desired, is set by Seed.

The Passband mode parameter is not used in version 0.32; it pertains to the manipulation of synthetic spectra and their multiplication with passband transmission functions. It will be relevant once on-the-fly passband integration is implemented.

Data may be binned; this may be desireable when the number of data points per light curve is very large, i.e. 10,000 or more. In that case it might prove useful to do most of the fitting on binned light curves and revert to a much more time-expensive computation on full data-sets once the model is close to the solution.

Finally, the radial velocity proximity effects for the primary and the secondary star are turned on by default, and can be turned off by unchecking the Primary and Secondary proximity effects checkboxes.

Once you are done setting the data, you may save the current setup into a parameter file. You do this by clicking on the **Save** button or selecting the option from the File menu. By convention, **PHOEBE** parameter files are given the extension .phoebe or .pho.

4.3 Data weighting

It is a good practice to use the panel **RV plot** or **LC plot** to display each data set just entered to be sure that it is what you want and that PHOEBE reads it correctly.

4.3 Data weighting

PHOEBE supports three types of weights.

Intrinsic weights. As a measure of per-observation precision, each data point can have an intrinsic weight attributed to it. For N data points there will be N intrinsic weights, which may be (optionally) supplied in the third column of the input file. These weights can be expressed either in terms of *standard weight* or in terms of *standard deviation*.

In the case of standard weight, the squares of the residuals are multiplied by the respective standard weights (cf. Eq. 1). Keep in mind that their absolute scaling bears no significance on minimization: it will only make the final cost function appear larger or smaller. This means that, if all individual weights across all passbands are multiplied by the same constant, the solution will be exactly the same. However, the relative scaling across passbands is very significant: if individual weights of a single curve were rescaled by a given constant, that passband would either gain or loose the relative weight in comparison to other curves. You must thus insure that all individual weights across all passbands are scaled uniformly.

In the case of standard deviation, the squares of the residuals are divided by the square of respective standard deviations (cf. Eq. 2). Every data point now has a measure of precision that is directly comparable to all other data points across all other passbands. Since standard deviation will by typically provided in magnitudes, PHOEBE transforms σ_m to σ_f ; this is a non-symmetric transformation ($\sigma_{f+} \neq \sigma_{f-}$) and PHOEBE uses a conservative approach of selecting the larger of the two.

Passband-dependent weights. Every light and radial velocity curve has a corresponding precision that is typically expressed in terms of standard deviation (or r.m.s.). Each curve should thus be weighted by its relative precision. If the intrinsic weights are given in terms of standard weight, a passband-dependent weight of that curve will then simply be the inverse square of its standard deviation, $w_p = 1/\sigma_p^2$. If, however, the intrinsic weights are given in terms of standard deviation, then these

observations are *already* weighted in the absolute sense and applying a passband-dependent weight would cause those points to be weighted twice. That is why, for w_i given in terms of standard deviation, w_p (and thus σ_p) should be set to 1.

Level-dependent weights. Both photometric and spectroscopic data acquisition is essentially photon counting, with a reasonably well understood underlying statistics of how the noise scales with the light level. In a Poisson regime (i.e. shot noise), the noise is proportional to the square root of the flux: $N \propto \sqrt{f}$. The S/N ratio thus increases with the square root of the increasing flux, but so does the resulting scatter; in turn, the level-dependent weight is inversely proportional to flux: $w \propto f^{-1}$. In a low light level regime (i.e. scintillation noise), the noise is linearly proportional to flux; the S/N ratio is constant with the increasing flux, so the resulting scatter increases linearly. The level-dependent weight in this regime is thus proportional to the inverse square of the flux: $w \propto f^{-2}$. If the data are dominated by sources of scatter other than those governed by the counting statistics (i.e. detector noise) and are thus largely independent of the light level, the S/N ratio will then increase linearly with the increasing flux, implying that the noise is constant and there should be no level-dependent weighting applied. Radial velocity observations are a most notable example of this regime; that is why level-dependent weighting applies only to light curves.

The cost function of the fit that takes all three sources of noise into account is expressed as:

$$\chi^2 = \sum_p \frac{1}{\sigma_p^2} \sum_{i=1}^{N_p} \frac{w_i (f_i - s_i)^2}{f_i^R},$$
(1)

where index p runs over all available passbands, index i runs over N_p observations for p-th passband, w_i are standard weights, f_i and s_i are observed and calculated fluxes, respectively, and R is the level-dependent regime power: R = 0 for no level-dependent weighting, R = 1 for Poisson noise, and R = 2for low light noise. If intrinsic weights are given in terms of standard deviation σ_i , Eq. (1) becomes:

$$\chi^2 = \sum_p \frac{1}{\sigma_p^{*2}} \sum_{i=1}^{N_p} \frac{(f_i - s_i)^2}{\sigma_i^2 f_i^R},$$
(2)

Note that, as mentioned above, σ_i assume the role of passband weighting as well, and σ_p^* need to be set to 1. It may sometimes be desireable to subjectively increase or decrease the influence of any given curve in the total value of χ^2 ; this is achieved by setting σ_p^* to a value different from 1: larger for less relative weight and smaller for more relative weight. That is the reason why we do not omit the term with σ_p^* in Eq. (2). In particular, σ_p^* should be almost exclusively 1 for all photometric curves, but it may be sometimes tweaked slightly for the radial velocity curves so that their relative contribution is similar to those of light curves.

PHOEBE provides four important values in the Statistics summary of the Fitting tab for each passband:

Unweighted residuals:	$\sum_{i} (f_i - s_i)^2$
Intrinsically weighted residuals:	$\sum_{i} w_i (f_i - s_i)^2$
Passband weighted residuals:	$1/\sigma_{p}^{2}\sum_{i}w_{i}(f_{i}-s_{i})^{2}$
Fully weighted residuals:	$1/\sigma_p^2 \sum_i w_i (f_i - s_i)^2 / f_i^R$

The cost function that is used to drive the minimizer is a sum of fully weighted residuals and is displayed in the Fitting tab.

4.4 Propagating formal errors

When modeling eclipsing binary light and radial velocity curves simultaneously, it is not always clear how to proceed with computing formal errors of the computed parameters such as stellar masses and radii. While we are working on fundamentally changing the approach that is currently used to compute formal errors (DC covariances will be replaced by Bayesian inference), we present here what seems to be best practice with the currently available modeling software. Since each eclipsing binary has a unique parameter space, with characteristic parameter correlations and degeneracy, your mileage may vary when following this "recipe".

TODO: Discuss LC-dominated and RV-dominated solutions

Computing formal errors for masses and radii basically boils down to properly propagating formal errors of adjusted parameters. Propagation can either be done the classical way (by propagating absolute or relative errors as appropriate – and you'll grow old trying to do this) or by evaluating a log of the expression, differentiating it, and approximating $\sigma_p \approx |dp|$ for all parameters p. Absolute value in |dp| assures that formal errors accumulate. Note that this is the most conservative approach, which can often be replaced with summing the squares of standard deviations rather than adding them linearly.

For masses we simply use Kepler's 3rd law, while for the radii we need

to use the expression for the generalized surface potential (Wilson 1979) for elliptical orbits and asynchronous rotation:

$$\Omega = \frac{1}{R} + q \left[\frac{1}{[R^2 + D^2 - 2R\lambda D]^{1/2}} - \frac{\lambda R}{D^2} \right] + \frac{1}{2} F^2 (1+q) R^2 (1-\nu^2), \quad (3)$$

where λ and ν are direction cosines, D is instantaneous separation of the two components, R is the fractional radius (relative to the semi-major axis), $q = M_2/M_1$ is the mass ratio, and F is synchronicity parameter.

TODO: Discuss the determination of formal errors from the covariance matrix

Once you have a final solution at hand, formal errors should be computed by following these steps.

- 1. Compute $a \sin i$;
- 2. using **only** RV curves, set $i = 90^{\circ}$, set a = x, and adjust the following parameters: $a, q, v_{\gamma}, \Delta \Phi, e$ and ω . Note that the fitted parameter ais a proxy to $a \sin i$. Record formal errors without adopting adjusted values of parameters. This way you should have obtained $\sigma_{a \sin i}, \sigma_q$ and $\sigma_{v_{\gamma}}$; discard formal errors of the remaining parameters for now, as they were only used to account for parameter correlations. Since light curves were not used, these errors have likely been overestimated anyway.
- 3. Set the semi-major axis and inclination back to their original values.
- 4. Using a full set of available light and RV curves, adjust all parameters that have been varied (i.e. not assumed or derived from the outside, i.e. spectroscopy) but *omitting* a, q and v_{γ} . Once the convergence is reached, record the formal errors of the adjusted parameters. Again, do not adopt the corrected values.
- 5. Take $\sigma_{a \sin i}$ from the RV fit and σ_i from the simultaneous fit and compute σ_a :

$$\sigma_a = \frac{\sigma_{a\sin i}}{\sin i} + \frac{a\cos i \,\sigma_i}{\sin i} \tag{4}$$

6. Compute radii by the Newton-Raphson method (or simply trust PHOEBE to do this for you). Since the shape of a star is fully determined by the surface potential Ω , Eq. (3) has to be iteratively inverted to obtain the radius R for the given λ and ν . This is most easily achieved by expressing the potential at the star's pole ($\lambda = 0, \nu = 1$):

$$\Omega = \frac{1}{R_{\text{pole}}} + \frac{q}{[D^2 + R_{\text{pole}}^2]^{1/2}}.$$
(5)

Asserting that Ω is constant over the surface of the star yields:

$$\frac{1}{R} + q \left(\frac{1}{[R^2 + D^2 - 2R\lambda D]^{1/2}} - \frac{R\lambda}{D^2} \right) + \frac{1}{2} F^2 (1+q) R^2 (1-\nu^2) =$$
$$= \frac{1}{R_{\text{pole}}} + q \left(\frac{1}{[D^2 + R_{\text{pole}}^2]^{1/2}} \right).$$
(6)

Solving this equation iteratively for R given any λ and ν determines the local fractional radius of the star.

7. Use σ_a , σ_P and σ_q to compute formal errors of individual masses:

$$M_{1} = \frac{4\pi^{2}a^{3}}{GP^{2}} \frac{1}{1+q}; \quad M_{2} = \frac{4\pi^{2}a^{3}}{GP^{2}} \frac{q}{1+q};$$

$$\sigma_{M_{1}} = M_{1} \left(3\frac{\sigma_{a}}{a} + 2\frac{\sigma_{P}}{P} + \frac{\sigma_{q}}{q+1} \right); \quad (7)$$

$$\sigma_{M_2} = M_2 \left(3\frac{\sigma_a}{a} + 2\frac{\sigma_P}{P} + \frac{\sigma_q}{q(q+1)} \right).$$
(8)

8. Use σ_{Ω} , σ_q , σ_e and σ_F to compute formal errors of fractional radii:

$$\sigma_R = \left| \frac{1}{A_1} \right| \sigma_\Omega + \left| \frac{A_2}{A_1} \right| \sigma_q + \left| \frac{A_3}{A_1} \right| \sigma_e + \left| \frac{A_4}{A_1} \right| \sigma_F, \tag{9}$$

where:

$$A_{1} = -\frac{1}{R^{2}} - \frac{q(R+e-1)}{[R^{2}+(1-e)^{2}-2R(1-e)]^{3/2}} - \frac{q}{(1-e)^{2}} + F^{2}(1+q)R,$$

$$A_{2} = \frac{1}{[R^{2}+(1-e)^{2}-2R(1-e)]^{1/2}} - \frac{R}{(1-e)^{2}} + \frac{1}{2}F^{2}R^{2},$$

$$A_{3} = -\frac{q(R+e-1)}{[R^{2}+(1-e)^{2}-2R(1-e)]^{3/2}} - \frac{2qR}{(1-e)^{3}},$$

$$A_{4} = F(1+q)R^{2}.$$
(10)

Here we assumed the point radius, i.e. $\lambda = 1$, $\nu = 0$, and periastron separation D = 1 - e, to get the largest estimate on σ_R .

5 SOLVING THE INVERSE TASK IN PHOEBE

5.1 DIFFERENTIAL CORRECTIONS

The choice of differential correction methods of consecutive iterations of the chosen physical parameters is the built-in initial choice in PHOEBE. Before running it, think a bit about the range of inidividual parameters and their steps, defined in panel PARAMETERS and modify them if necessary to suite to your task. As explained by Prša and Zwitter (2006), convergency of primary flux levels from the beginning along with other parameters leads to a very slow oscillatory convergence. It is therefore very desirable to go first to PARAMETERS/LUMINOSITIES and use the option CALCULATE step by step for each data subset. Only then start the convergency of all desired parameters using the top panel FITTING. Andrej, please explain how to decide that convergency was reached after some number of iterations and provide also other advice on the strategy of the solution.

5.2 NELDER AND MEAD SIMPLEX

Andrej, just questions here: Please, explain whether one should ask for calculation of the cost function right before running the fitting in the internal panel FITTING. Again - when the convergency was reached? Why the cost function is different from that calculated by differential corrections? How to prevent the program to run out of permissible range?

5.3 USING THE SCRIPTER

Andrej, it would be great to put here a few examples of batches of scripter commands for some typical calculations. If fixed format is needed soemwhere, you can use the sample Table attached below after 'end of document' command.

6 OUTPUT OF RESULTS

1. Note that various pieces of information about the results are scattered over various panels. Thus, for instance all physical epochs for elliptical orbit are listed in panel PARAMETERS/ORBIT.

- 2. Note that masses, radii, log g and bolometric magnitudes are only upgraded when you run LC PLOT. It is so because only this plot actives calculation with the LC program.
- 3. In panel **Fitting** one gets lines with information about individual data sets. The "Relative weight" column has been changed to "Weighted sum of residuals". The values reported there are:

$$\frac{1}{\sigma_i^2} \sum_i w_i w_p (c_i) (o_i - c_i)^2 \tag{11}$$

where σ_i is the passband sigma, w_i are individual weights, and $w_p(c_i)$ are level-dependent weights.

4. Be aware that phasing the light curves in PHOEBE does not take apsidal motion into account, i.e. only dP/dt is considered in the calculation of phases, while $d\omega/dt$ is not. The reason is that the apsidal motion is actually changing the shape of the light and RV curves and the phase separation of the primary and secondary eclipses. Therefore, if one wants to see how good the fit is, the only sensible option is to display residuals vs. time (HJD).

It would be great to include into this manual a table with individual parameters which can be converged with indication in which units they are treated. Important especially for paramaters such a period derivative etc. Would it be too complicated to have ω and $\dot{\omega}$ in degrees and degs. per day, respectively, on the input and output as used in the vast majority of publications? Could optionally be the luminosities in the output be normalized i.e. L1+L2+L3=1? For larger number of datasets one has to do lot of calculations manually and the advantage of these relative luminosities is that you see immediately whether they are changing in a logical way with wavelength.

References

Castelli, F. and Kurucz, R. L.: 2004, ArXiv Astrophysics e-prints

van Hamme, W.: 1993, AJ 106, 2096

Wilson, R. E. and Devinney, E. J.: 1971, ApJ 166, 605